$\tan x = \dfrac{\text{sen } x}{\cos x}$

$\cot x = \dfrac{\cos x}{\text{sen } x}$

$\sec x = \dfrac{1}{\cos x}$

$\csc x = \dfrac{1}{\text{sen } x}$

$\text{sen}^2 x + \cos^2 x = 1$

$\text{sen}^2 x = 1 - \cos^2 x \Rightarrow$

$\cos^2 x = 1 - \text{sen}^2 x$

$\sec^2 x - \tan^2 x = 1 \Rightarrow$

$\csc^2 x - \cot^2 x = 1$

En los ejercicios 7 a 55 pruebe las identidades dadas:

7. $\dfrac{\text{sen } x + \cos x}{\text{sen } x} = 1 + \dfrac{1}{\tan x}$

Solución:

$$\frac{\text{sen } x + \cos x}{\text{sen } x} = 1 + \frac{1}{\tan x} \Leftrightarrow \frac{\text{sen } x + \cos x}{\text{sen } x} = 1 + \cot x,$$

$$\frac{\text{sen } x + \cos x}{\text{sen } x} = 1 + \frac{1}{\tan x} \Leftrightarrow \frac{\text{sen } x + \cos x}{\text{sen } x} = 1 + \frac{\cos x}{\text{sen } x},$$

$$\frac{\text{sen } x + \cos x}{\text{sen } x} = 1 + \frac{1}{\tan x} \Leftrightarrow \frac{\text{sen } x + \cos x}{\text{sen } x} = \frac{\text{sen } x + \cos x}{\text{sen } x}.$$

9. $\dfrac{\text{sen } x}{\csc x} + \dfrac{\cos x}{\sec x} = 1$

Solución:

$$\frac{\text{sen } x}{\csc x} + \frac{\cos x}{\sec x} = 1 \Leftrightarrow \frac{\dfrac{\text{sen } x}{1}}{\dfrac{1}{\text{sen } x}} + \frac{\dfrac{\cos x}{1}}{\dfrac{1}{\cos x}} = 1,$$

$$\Rightarrow \quad \frac{\text{sen } x}{\csc x} + \frac{\cos x}{\sec x} = 1 \Leftrightarrow \text{sen}^2 x + \cos^2 x = 1;$$

$$\therefore \quad \frac{\text{sen } x}{\csc x} + \frac{\cos x}{\sec x} = 1 \Leftrightarrow 1 = 1.$$

11. $\dfrac{\sec y}{\tan y + \cot y} = \text{sen } y$

Solución:

$$\frac{\sec y}{\tan y + \cot y} = \text{sen } y \Leftrightarrow \frac{\dfrac{1}{\cos y}}{\dfrac{\text{sen } y}{\cos y} + \dfrac{\cos y}{\text{sen } y}} = \text{sen } y \Leftrightarrow \frac{\dfrac{1}{\cos y}}{\dfrac{\text{sen}^2 y + \cos^2 y}{\text{sen } y . \cos y}} = \text{sen } y,$$

$$\Rightarrow \quad \frac{\sec y}{\tan y + \cot y} = \text{sen } y \Leftrightarrow \frac{\dfrac{1}{\cos y}}{\dfrac{1}{\text{sen } y . \cos y}} = \text{sen } y \Leftrightarrow \frac{\text{sen } y . \cos y}{\cos y} = \text{sen } y;$$

$$\therefore \quad \frac{\sec y}{\tan y + \cot y} = \text{sen } y \Leftrightarrow \text{sen } y = \text{sen } y.$$

13. $\dfrac{1-\operatorname{sen} x}{\cos x}=\dfrac{\cos x}{1+\operatorname{sen} x}$

Solución:

$$\frac{1-\operatorname{sen} x}{\cos x}=\frac{\cos x}{1+\operatorname{sen} x} \Leftrightarrow (1-\operatorname{sen} x)(1+\operatorname{sen} x)=\cos x \cdot \cos x \Leftrightarrow 1-\operatorname{sen}^2 x=\cos^2 x;$$

$$\therefore \quad \frac{1-\operatorname{sen} x}{\cos x}=\frac{\cos x}{1+\operatorname{sen} x} \Leftrightarrow \cos^2 x=\cos^2 x.$$

18. $\dfrac{\tan x-\operatorname{sen} x}{\operatorname{sen}^3 x}=\dfrac{\sec x}{1+\cos x}$

MathType 5.0 Equation

Solución:

$$\frac{\tan x-\operatorname{sen} x}{\operatorname{sen}^3 x}=\frac{\sec x}{1+\cos x} \Leftrightarrow \frac{\frac{\operatorname{sen} x}{\cos x}-\operatorname{sen} x}{\operatorname{sen}^3 x}=\frac{\sec x}{1+\cos x},$$

$$\Rightarrow \quad \frac{\tan x-\operatorname{sen} x}{\operatorname{sen}^3 x}=\frac{\sec x}{1+\cos x} \Leftrightarrow \frac{\frac{\operatorname{sen} x-\operatorname{sen} x\cos x}{\cos x}}{\operatorname{sen}^3 x}=\frac{\sec x}{1+\cos x},$$

$$\Rightarrow \quad \frac{\tan x-\operatorname{sen} x}{\operatorname{sen}^3 x}=\frac{\sec x}{1+\cos x} \Leftrightarrow \frac{\frac{\operatorname{sen} x(1-\cos x)}{\cos x}}{\operatorname{sen}^3 x}=\frac{\sec x}{1+\cos x} \Leftrightarrow \frac{1-\cos x}{\operatorname{sen}^2 x\cos x}=\frac{\sec x}{1+\cos x}$$

$$\Rightarrow \quad \frac{\tan x-\operatorname{sen} x}{\operatorname{sen}^3 x}=\frac{\sec x}{1+\cos x} \Leftrightarrow \frac{1-\cos x}{(1-\cos^2 x)\cos x}=\frac{\sec x}{1+\cos x},$$

$$\frac{\tan x-\operatorname{sen} x}{\operatorname{sen}^3 x}=\frac{\sec x}{1+\cos x} \Leftrightarrow \frac{(1-\cos x)}{(1-\cos x)(1+\cos x)\cos x}=\frac{\sec x}{1+\cos x},$$

$$\frac{\tan x-\operatorname{sen} x}{\operatorname{sen}^3 x}=\frac{\sec x}{1+\cos x} \Leftrightarrow \frac{1}{(1+\cos x)\cos x}=\frac{\frac{1}{\cos x}}{1+\cos x};$$

tgA + 2cosA cscA = secA cscA + ctgA

(senA / cosA) + 2cosA (1/senA) = [sen^2A + 2$\cos^2$A]/(senA cosA) =

15. $\sec x(1-\operatorname{sen}^2 x)=\cos x$

Solución:

$$\sec x(1-\operatorname{sen}^2 x)=\cos x \Leftrightarrow \frac{1}{\cos x}(\cos^2 x)=\cos x;$$

MathType 5.0 Equation

$$\therefore \quad \sec x(1-\operatorname{sen}^2 x)=\cos x \Leftrightarrow \cos x=\cos x.$$

16. $\tan z\cdot\cos z\cdot\csc z=1$

Solución:

$$\tan z\cdot\cos z\cdot\csc z=1 \Leftrightarrow \frac{\operatorname{sen} z}{\cos z}\cdot\cos z\cdot\frac{1}{\operatorname{sen} z}=1 \Leftrightarrow \frac{\cancel{\operatorname{sen} z}}{\cancel{\cos z}}\cdot\cancel{\cos z}\cdot\frac{1}{\cancel{\operatorname{sen} z}}=1;$$

$$\therefore \quad \tan z\cdot\cos z\cdot\csc z=1 \Leftrightarrow 1=1.$$

45. $\dfrac{\tan x+\cot x}{\tan x-\cot x}=\dfrac{\sec^2 x}{\tan^2 x-1}$

Solución:

$$\frac{\tan x+\cot x}{\tan x-\cot x}=\frac{\sec^2 x}{\tan^2 x-1}\Leftrightarrow\frac{\dfrac{\operatorname{sen} x}{\cos x}+\dfrac{\cos x}{\operatorname{sen} x}}{\dfrac{\operatorname{sen} x}{\cos x}-\dfrac{\cos x}{\operatorname{sen} x}}=\frac{\dfrac{1}{\cos^2 x}}{\dfrac{\operatorname{sen}^2 x}{\cos^2 x}-1}$$

$$\Leftrightarrow\frac{\dfrac{\operatorname{sen}^2 x+\cos^2 x}{\operatorname{sen} x\cos x}}{\dfrac{\operatorname{sen}^2 x-\cos^2 x}{\operatorname{sen} x\cos x}}=\frac{\dfrac{1}{\cos^2 x}}{\dfrac{\operatorname{sen}^2 x-\cos^2 x}{\cos^2 x}}\Leftrightarrow\frac{\operatorname{sen}^2 x+\cos^2 x}{\operatorname{sen}^2 x-\cos^2 x}=\frac{1}{\operatorname{sen}^2 x-\cos^2 x}$$

$$\Leftrightarrow\frac{1}{\operatorname{sen}^2 x-\cos^2 x}=\frac{1}{\operatorname{sen}^2 x-\cos^2 x}.$$

(tgA + ctgA)(cosA + senA) = cscA + secA

[(senA / cosA) + (cosA / senA)](cosA + senA) = [(sen^{2}A + cos^{2}A)/(senA cosA)](cosA + senA) =

[1/(senA cosA)](cosA + senA) = cosA / (senA cosA) + senA / (senAcosA) = 1/senA + 1/cosA =

cscA + secA

tg^{2}A – sen^{2}A = tg^{2}A sen^{2}A

(sen^{2}A / cos^{2}A – sen^{2}A) = sen^{2}A [(1/cos^{2}A) – 1] = sen^{2}A (1 – cos^{2}A)/cos^{2}A =

sen^{2}A sen^{2}A / cos^{2}A = sen^{2}A tg^{2}A

(secA – tgA)(cscA + 1) = ctgA

[(1/cosA) – senA/cosA][1/senA + 1] = [(1 – senA)/cosA][(1 + senA)/senA] =

(1 – sen^{2}A)/[senA cosA] = cos^{2}A / [senA cosA] = cosA / senA = ctgA

(1 – senA)(secA + tgA) = cosA

(1 – senA)(1/cosA + sen/cosA) = (1 – senA)[1 + senA]/cosA = (1 – sen^{2}A)/cosA = cos^{2}A/cosA = cosA

senA /(1 – cosA) = cscA + ctgA

[senA (1 + cosA)] / [(1 – cosA)(1 + cosA)] = (senA + senA cosA)/(1 – cos^{2}A) =

$(senA + senA\ cosA)/sen^2A = senA/sen^2A + senAcosA/sen^2A = (1/senA) + cosA/senA = cscA + ctgA$

tgA + 2cosA cscA = secA cscA + ctgA

$(senA / cosA) + 2cosA\ (1/senA) = [sen^2A + 2cos^2A]/(senA\ cosA) =$
$[sen^2A + cos^2A + cos^2A]/(senA\ cosA) = (1 + cos^2A)/(senA\ cosA) =$
$1/(senA\ cosA) + cos^2A / (senA\ cosA) = cscA\ secA + ctgA$

(tgA + ctgA)(cosA + senA) = cscA + secA

$[(senA / cosA) + (cosA / senA)](cosA + senA) = [(sen^2A + cos^2A)/(senA\ cosA)](cosA + senA) =$
$[1/(senA\ cosA)](cosA + senA) = cosA / (senA\ cosA) + senA / (senAc$

Identidades Trigonométricas Fundamentales:

1.- $tg\alpha = \frac{sen\alpha}{\cos\alpha}$

2.- $\sec\alpha = \frac{1}{\cos\alpha}$

3.- $c\sec\alpha = \frac{1}{sen\alpha}$

4.- $ctg\alpha = \frac{1}{tg\alpha} = \frac{\cos\alpha}{sen\alpha}$

5.- $sen^2\alpha + \cos^2\alpha = 1$

6.- $tg^2\alpha + 1 = \sec^2\alpha$

7.- $ctg^2\alpha + 1 = c\sec^2\alpha$

$sen\alpha\ \ cosec\alpha = 1$

$cos\alpha\ \ sec\alpha = 1$

$tang\ \alpha\ \ cotg\alpha = 1$

a) $Ctg\ x\ Sen\ x \cong Cos\ x$	**b)** $Sen\ y\ Sec\ y \cong Tag\ y$	**c)** $\frac{Tag\ x}{Sen\ x} \cong Sec\ x$
d) $Sec^2x\ Ctg^2x \cong Csc^2x$	**e)** $\frac{Cosx + Cotg\ x}{1 + \cos c\ x} \cong \cos x$	**f)** $Sec^2 x \cong Cosc\ x\ Sen\ x + \frac{1}{Ctg^2 x}$
h) $\frac{Sen\ x}{Cos\ x} + \frac{Cos\ x}{Sen\ x} = \frac{Sec\ x}{Sen\ x}$	**i)** $Tag\ x + \frac{1}{Tag\ x} \cong \frac{Sec\ x}{Sen\ x}$	**j)** $Tag\ x + Ctg\ x \cong Sec\ x\, Csc\ x$
k) $2Sec\ x\ Ctg\ x \cong 2Csc\ x$	**l)** $Sec\ A - Tag\ A\ Sen\ A \cong Cos\ A$	***m)*** $(Sen\ x + Cos\ x)^2 \cong 2Sen\ x\ Cos\ x + 1$
ñ) $\frac{Sen\ x + Tag\ x}{1 + Sec\ x} \cong Sen\ x$	**o)** $Csc^2x \cong \frac{1}{1 - Cos^2x}$	***p)*** $\frac{Sen\ x}{1 + Cos\ x} + \frac{Cos\ x}{Sen\ x} \cong Csc\ x$
q) $Sen\ x(Csc\ x - Sec\ x) \cong 1 - Tag\ x$	**r)** $Sec^2 x - Sen^2 x = \cong Cos^2x + Tag^2x$	**s)** $(Sec^2x - 1)Ctg^2x \cong 1$
t) $Sec^2x(1 - Sen^2x) \cong 1$	**v)** $Cos^2x - Sen^2x \cong 2Cos^2x - 1$	**w)** $(1 + Ctg^2x)Sen^2x \cong 1$
y) $1 - Tag^2A \cong 2 - Sec^2A$	**z)** $\frac{Sec\ x\ Ctg\ x}{Csc^2x} \cong Sen\ x$	**aa)** $\frac{Cos\ x\ Sec\ x}{Tag\ x} \cong Ctg\ x$

ab) $(1+Tag^2 A)(1-Cos^2 A) \cong Tag^2 A$	**ac)** $\frac{Sec\,x}{Cos\,x}-\frac{Tag\,x}{Ctg\,x} \cong 1$	***ad)*** $\frac{1+Ctg^2 y}{1+Tag^2 y} \cong Ctg^2 y$
ae) $(Ctg\,A+1)^2+(Ctg\,A-1)^2 \cong 2\,Csc^2\,A$	ad) $1+\cot^2 x \cong \cos c^2 x$	ah) $(\sec x+1)(\sec x-1) \cong \tan g^2 x$
ai) $(Ctg\;x+\tan gx)^2 \cong Csc^2 x+\sec^2 x$	aj) $\frac{\cos^2 x-\tan g^2 x}{sen^2\,x} \cong \cot g^2 x-\sec^2 x$	ak) $\frac{sen\,2x}{sen\,x}-\frac{\cos 2x}{\cos x} \cong \sec x$